Intensity measure transformation of fragility curves for 2D buildings using simplified models

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Abstract: Seismic fragility curves are an essential tool for any risk assessment endeavour. While there is a wealth of studies that have provided high quality fragilities for many different types of structures, these curves are typically presented in terms of a single intensity measure (IM). To keep using such valuable data, an analyst is either forced to adopt the same (potentially suboptimal) IM, or completely discard them and restart with a new one. Instead, we propose a simple method for transforming a fragility curve to any IM of choice by using an equivalent single-degree-of-freedom model and its incremental dynamic analysis results to disaggregate the fragility to its constituent record-level results. Validation results from two complex 2D building hint that there is promise to this approach, offering nearly-error-free transformations of global-response fragilities at the cost of a few response history analyses of a nonlinear oscillator.

Keywords: seismic fragility, equivalent single-degree-of-freedom, intensity measure, building structures

1. Introduction

Fragility curves often provide a useful tool for a performance-based assessment for buildings of different characteristics by offering the probability of a structure exceeding a certain level of damage (Bakalis and Vamvatsikos 2018; Silva et al. 2019). Many literature studies provide seismic fragility curves that can be exploited by researchers for risk assessment studies; they are either analytically derived based on structural analyses (Erberik 2008; Kappos and Panagopoulos 2010; Rossetto and Elnashai 2005), or fitted directly on empirical data from past observations (Giordano et al. 2021; Rosti et al. 2021; Rossetto and Elnashai 2003). Often though, these curves do not provide the flexibility needed to be effectively used in a wider setting. The most prominent problem is that the fragility curves provided use a specific intensity measure (IM), often this being the peak ground acceleration (PGA) or the spectral acceleration at a given period. In several cases, recent literature may recommend other intensity measure definitions as more suitable for a reliable assessment of a building's behaviour (e.g., Kazantzi and Vamvatsikos 2015), but regardless of that, the IM used in each study often makes its results unavailable to another investigator using a different IM. This makes the valuable information encoded in fragility curves essentially useless outside of the specifics of their original source. Being able to transform one IM to another would at least resolve this basic incompatibility. Theoretically, having the full analysis or observational data on which a fragility is based would allow an easy transformation. Lacking such a solid foundation, an alternative path towards a potential solution could be the development of an equivalent model that would require only some of the structure's basic structural characteristics to allow us to disassemble or disaggregate the original curves back into a facsimile of the original data that they were based on. In this way, the transformation between different IMs becomes possible. As the simplest such model, we shall herein investigate the suitability of an elastic-perfectly-plastic single degree of freedom (SDOF) model.

2. Methodological approach

As a simple application scenario, let us assume we have a multi-degree-of-freedom (MDOF) structure (but not necessarily its model) and its fragility curves for one or more limit-states, which we want to transform from their original IM to a new one. As a first step, an equivalent single-degree-of-freedom (ESDOF) model is developed, using the original MDOF characteristics (Fig.1), e.g. the structure's fundamental eigenperiod and an estimate of its base shear strength, allowing for a bilinear representation of its capacity curve, as typical in nonlinear static procedures (e.g. EN1998-3, ASCE 41-17). This ESDOF is then subjected to incremental dynamic analysis (IDA, Vamvatsikos and Cornell 2002) using a single set of records. The record set of the original structure's study may be adopted, if available, a record selection process could be employed if one prefers to have hazard consistency with the site of interest, or a generic site-agnostic set may be chosen as a last resort.

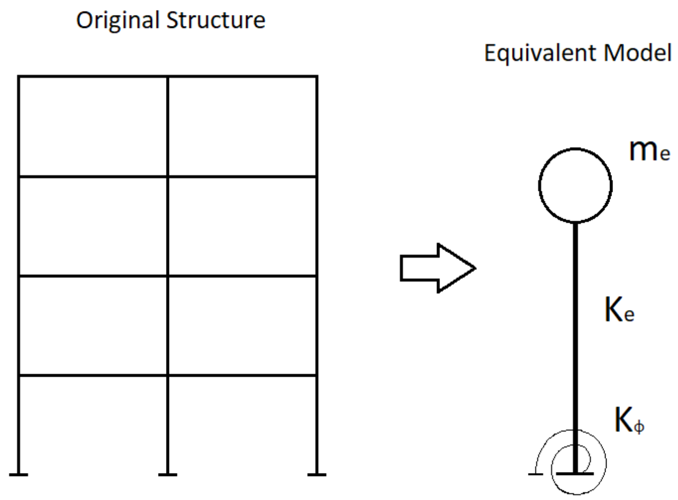

Fig. 1 – Equivalent single-degree-of-freedom cantilever model definition.

The first hurdle is estimating the engineering demand parameter (EDP) threshold for the ESDOF, EDP_{lim}^{ESDOF}, which would produce the "same" fragility in the ESDOF, as the threshold of EDP_{lim}^{MDOF} would produce in the full model. For example, if the EDP of interest is the roof drift for a threshold in the elastic region of response, then one could state that $EDP_{lim}^{ESDOF} = EDP_{lim}^{MDOF}/\Gamma$, where Γ is the first-mode participation factor. In general, though, more complex EDPs may come into play, involving interstory drift, floor accelerations or member moments and forces, which may be tough to predict a priori. Furthermore, there can be cases where we are only supplied with the MDOF fragility but not the limiting EDP value that accompanies it. Thus, we shall use such simplified estimates only as a seed value, or $EDP_{lim,0}^{ESDOF}$, in an iterative approach to determine a near-optimal "corresponding" value for the ESDOF. Thus, if m_{old}^{MDOF} and β_{old}^{MDOF} are the median and dispersion, respectively, of the original MDOF lognormal fragility in terms of the original "old" IM, then the seed estimate of $EDP_{lim,0}^{ESDOF}$ threshold is employed to determine a new candidate fragility. Say that $m^{ESDOF}_{old,i-1}$, $\beta^{ESDOF}_{old,i-1}$ (where *i* = 1 initially) are its corresponding parameters in terms of the old IM. Then the α_i factor is defined as the ratio of the MDOF over the ESDOF median for the *i*-th iteration:

$$\alpha_i = m^{MDOF}_{old} / m^{ESDOF}_{old,i-1} \tag{1}$$

α_i is essentially a linear correction factor, meant to adjust the median parameter of the lognormal fragility of ESDOF, by multiplying α_i with the previously defined $EDP^{ESDOF}_{lim,i-1}$ through the following equation:

$$EDP^{ESDOF}_{lim,i} = \alpha_i \cdot EDP^{ESDOF}_{lim,i-1} \tag{2}$$

It essentially works by exploiting the equal displacement rule, valid for moderate and long period structures to adjust the EDP threshold in increasingly smaller steps by translating IM differences to EDP ones. Obviously, where the rule does not hold, other iterative algorithms should be employed, such as Newton-Raphson or a simple bisection.

Simultaneously, another correction process is at play, through the implementation of an additional external dispersion parameter $\beta_{a,i}$, defined as:

$$\beta_{a,i} = \begin{cases} \sqrt{{\beta_{a,i-1}}^2 + ({\beta^{MDOF}_{old}}^2 - {\beta^{ESDOF}_{old,i-1}}^2)} & \text{if } \beta^{MDOF}_{old} > \beta^{ESDOF}_{old,i-1} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

This parameter adjusts the dispersion for the fragility results calculated for the ESDOF to match the normally higher dispersion of the original fragility, calculated for the MDOF model. In the rare occasion that the dispersion of the ESDOF model happens to be higher than the one calculated for the MDOF, a bad EDP seed value would typically be the culprit, e.g. searching for an inelastic MDOF limit-state in the elastic or near-elastic ESDOF range. This extra variability parameter is simply used to update the ESDOF fragility dispersion via a squared-root-sum-of-squares rule:

$$\beta^{ESDOF}_{old,i} \leftarrow \sqrt{{\beta^{ESDOF}_{old,i}}^2 + {\beta_{a,i}}^2} \tag{4}$$

Through this iterative process $EDP^{ESDOF}_{lim,i}$ is sequentially adjusted until the ESDOF fragility values converge to the MDOF ones. In each iteration, in order to assess whether or not the matching of those fragilities to the target ones is satisfactory, the Root Mean Squared Error ($RMSE_i$) value is calculated over a given discretization of the probability space to examine, given a tolerance value (e.g. 1%), the overall difference between the MDOF fragilities and the ones calculated for the ESDOF model in terms of the initial IM_{old} used.

The determination of the "converged" value of $EDP^{ESDOF}_{lim,i}$ can now be used to disaggregate the old IM fragilities through the IDA results of the ESDOF, breaking them up into the *N* records used for the analysis. Specifically, the determination of the fragilities for the old IM on the ESDOF was done by virtue of vertical statistics, associating a single $IM_{old,j}$ value to each *j*-th record (*j* = 1…*N*). The empirical CDF of said values is essentially the ESDOF fragility curve in IM_{old}. Since these approximately reproduce the MDOF fragility, it follows that if one transforms each of them into the new IM, it should offer a good approximation of the MDOF fragility in this new space. To do so, a transformation factor C_j is defined as the ratio of the $IM_{new,us,j}$ and $IM_{old,us,j}$ values estimated for each of the unscaled records:

$$C_j = IM_{new,us,j} / IM_{old,us,j} \tag{5}$$

Then, the N old IM values estimated for the fragility in question can be converted to new IM coordinates as:

$$IM_{new,j} = IM_{old,i,j} \cdot C_j \quad (6)$$

Fitting a lognormal fragility to the $IM_{new,j}$ values allows the determination of its parameter, m_{new}^{ESDOF} and β_{new}^{ESDOF} that are now in terms of the "new" IM that was selected.

To account for the missing dispersion observed in the old IM space, the additional dispersion parameter of $\beta_{a,i}$, should be employed to augment the transformed fragilities:

$$\beta_{new}^{ESDOF} \leftarrow \sqrt{{\beta_{new}^{ESDOF}}^2 + {\beta_{a,i}}^2} \quad (7)$$

A detailed pseudo-algorithm representing this methodological process appears in Table 1.

Table 1: "TransformFragility" pseudo-algorithm for fragility IM transformation

 # Initialization
Form ESDOF model
Run IDA of ESDOF
Determine approximate $EDP_{lim,0}^{ESDOF}$ that corresponds to EDP_{lim}^{MDOF}
Estimate $IM_{\ old,0,j}$ (j=1…N) value that corresponds to $EDP_{lim,0}^{ESDOF}$ for each record j
Estimate lognormal fragility parameters, $m_{old,0}^{ESDOF}$ and $\beta_{old,0}^{ESDOF}$, corresponding to $IM_{\ old,0,j}$
Calculate $RMSE_0$ of ESDOF and MDOF fragility
$i\ =\ 0$
$\alpha_0 =\ 1.0$
$\sigma_{\alpha,0} =\ 0.01$

 # Iterative matching of ESDOF and MDOF fragilities
Repeat
  $i = i + 1$
  $\alpha_i = m_{old}^{MDOF}/m_{old,i-1}^{ESDOF}$
  $EDP_{lim,i}^{ESDOF} = \alpha_i \cdot EDP_{lim,i-1}^{ESDOF}$
  If $\beta_{old,i-1}^{ESDOF} < \beta_{old}^{MDOF}$
    $\beta_{a,i} = \sqrt{{\beta_{a,i-1}}^2 + ({\beta_{old}^{MDOF}}^2 - {\beta_{old,i-1}^{ESDOF}}^2)}$
  else
    $\beta_{a,i} = 0$
  end
  Estimate $IM_{\ old,i,j}$ (j = 1…N) value that corresponds to $EDP_{lim,i}^{ESDOF}$ for each record j
  Estimate lognormal fragility parameters, $m_{old,i}^{ESDOF}$ and $\beta_{old,i}^{ESDOF}$, corresponding to $IM_{\ old,i,j}$
  $\beta_{old,i}^{ESDOF} \leftarrow \sqrt{{\beta_{old,i}^{ESDOF}}^2 + {\beta_{a,i}}^2}$
  Calculate $RMSE_i$ of ESDOF and MDOF fragility
Until $(RMSE_i - RMSE_{i-1})/RMSE_{i-1} < tol$

 # Transform the fragility to a new IM
For j = 1 to N records
  Estimate $IM_{new,us,j}$ and $IM_{old,us,j}$ for unscaled record j
  $C_j = IM_{new,us,j}/\ IM_{old,us,j}$
  $IM_{new,j} = IM_{old,i,j} \cdot C_j$
end
Estimate lognormal fragility parameters, m_{new}^{ESDOF} and β_{new}^{ESDOF}, corresponding to $IM_{new,i,j}$
$\beta_{new}^{ESDOF} \leftarrow \sqrt{{\beta_{new}^{ESDOF}}^2 + {\beta_{a,i}}^2}$

3. Case studies

To examine the efficiency of the process presented in the previous paragraph, two typical 2D building models were used as validation examples, namely a 4-storey reinforced concrete (RC) building (Chatzidaki and Vamvatsikos 2021), and a 20-storey steel building (Lachanas and Vamvatsikos 2021). The goal is to employ the available MDOF models from the aforementioned publications to estimate fragilities in Sa(T_1) and attempt to transform them to PGA via the proposed procedure. By having the original IDA MDOF results available, we shall also be able to calculate the actual PGA fragilities and verify (or not) our approach.

For the typical 4-storey RC building, an ESDOF model was developed using the building's fundamental eigenperiod of T_1 = 1.05sec, and its pushover curve, as the defining features of the model. To perform the pushover analysis a first-mode load pattern was utilized for the initial building, and then, as shown in Fig. 2, a bilinear curve was fitted. The bilinear approach selected was the 10% fit described in De Luca et al. (2013). For the 20-storey building a similar procedure was employed with the definition of the equivalent model being based on the building's eigenperiod, T_1 = 3.82sec, and its pushover curve as illustrated in Fig. 2, using again the aforementioned bilinear approach. Since the bilinear approach for the pushover curves was adopted for the model definitions, the plastic behaviour of the model is taken into account using a rotational spring (K_Φ) at its base, with a rotational stiffness that would result to the corresponding pushover curve, simulating an elastic-perfectly-plastic behaviour identical to that illustrated in Fig. 2 for both buildings.

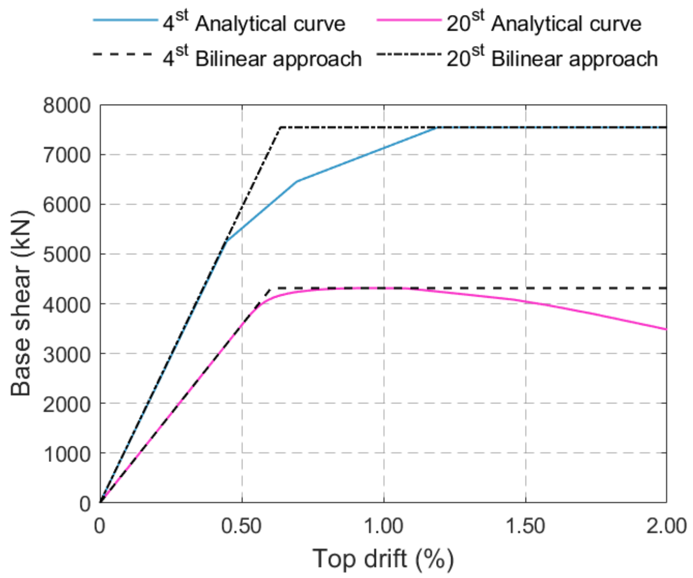

Fig. 2 – Pushover curves for the 4-storey and 20-storey buildings.

To calculate the fragility curves for the original buildings, IDA was performed using as an EDP, the Top Drift (TDR) of the structure. Using those analyses results and considering two limit states (LS) or capacity thresholds representing the EDP_{lim}^{MDOF} values, two LS cases were defined. Therefore, two fragility curves were calculated for each structure both in terms of spectral acceleration Sa(T_1), and PGA. The fragility curves for the buildings studied appear in Table 2, referred as the 'Original Structures (MDOF)' and were calculated for LS1: TDR = 0.75% and LS2: TDR = 2.0%. The same procedure was employed for the corresponding ESDOF models defined, with the record sets used for the analyses of the simplified models, being the same used for each of the original buildings. Given the initial fragility results for the simulated models ($m_{old,0}^{ESDOF}$, $\beta_{old,0}^{ESDOF}$) (Table 2), and the target fragilities calculated using the original models (m_{old}^{MDOF} and β_{old}^{MDOF}) in terms of IM_{old} = Sa(T_1), the process described in Table 1 was employed, for the adjustment of the calculated fragilities of the ESDOF models in terms of Sa(T_1), to the corresponding results of the MDOF detailed models ($m_{old,i}^{ESDOF}$, $\beta_{old,i}^{ESDOF}$), and the IM transformation to the IM_{new} = PGA (m_{new}^{ESDOF} , β_{new}^{ESDOF}). The lognormal fragility parameters calculated for each case are presented in Table 2.

In Fig. 3–4 a condensed overview of the results can be observed, in terms of simulating the fragility results of the original models (MDOF) with the simplified models discussed (ESDOF). The results are deemed to be satisfactory, given how close the simulated PGA and *Sa* fragilities are to the MDOF ones, with only minor errors observed for the transformed PGA case.

Table 2. Fragility curves parameters for all models.

Fragility Results			DS1		DS2	
			median (g)	dispersion	median (g)	dispersion
4-storey Building	Original Structure (MDOF)	$S_a(T_1)$	0.334	0.13	0.792	0.27
		PGA	0.352	0.43	0.829	0.42
	Seed Model (ESDOF,$_0$)	$S_a(T_1)$	0.429	0.09	1.101	0.29
		PGA	0.453	0.41	1.161	0.45
	Adjusted Model (ESDOF,$_i$)	$S_a(T_1)$	0.334	0.13	0.795	0.27
		PGA	0.352	0.43	0.838	0.45
20-storey Building	Original Structure (MDOF)	$S_a(T_1)$	0.112	0.25	0.304	0.33
		PGA	0.634	0.81	1.728	0.82
	Seed Model (ESDOF,$_0$)	$S_a(T_1)$	0.146	0.10	0.429	0.33
		PGA	0.826	0.86	2.431	0.88
	Adjusted Model (ESDOF,$_i$)	$S_a(T_1)$	0.113	0.28	0.308	0.31
		PGA	0.637	0.92	1.758	0.87

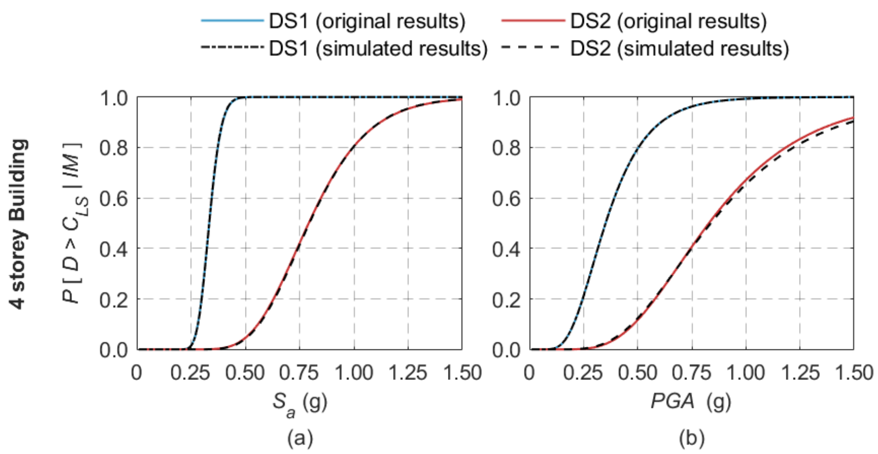

Fig. 3 – Fragility curves results for the 4-storey building.

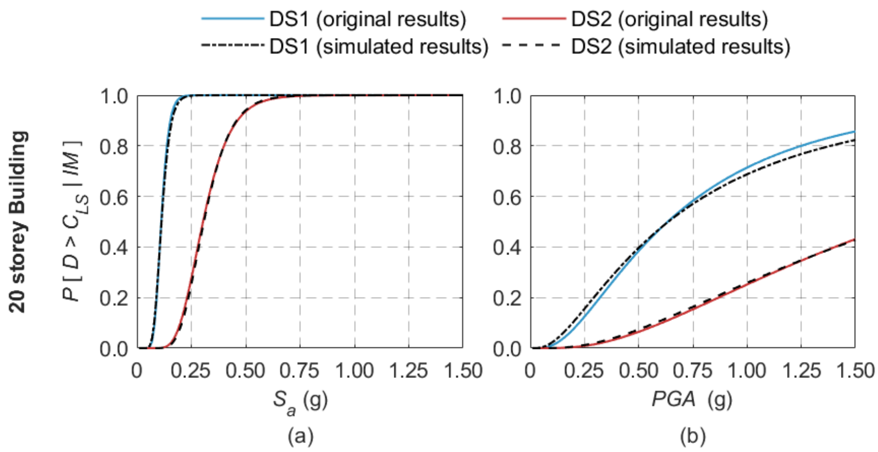

Fig. 4 - Fragility curves results for the 20-storey building.

4. Conclusions

A simple methodology for the intensity measure (IM) transformation of fragility curves was presented. Two building models were used as validation achieving good accuracy both for a high-rise 20-story and a low-rise 4-story frame. It was showcased that even though a single-degree-of-freedom model was proposed, this conversion should not be critically affected by its simplicity by virtue of an iterative correction process. Therefore, the proposed method could provide the required flexibility to transform any fragility curves to the desired IM, with a minimal error. Of course, further study is required in terms of quantifying the effects of the spectral shape of the seismic records selected for the simplified analysis, if the original records are not available. Moreover, the applicability to limit-states based on more localized measures of response, such as the interstorey drift or peak floor accelerations, should not be taken for granted, since they would be more difficult to capture using a model that discards the effect of higher modes.

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